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# Macro-Economic Equilibrium and Price Signal Accuracy in Non-Scalar Economy

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## Abstract:

*The management of global commons is of vital importance for modern society. The current solutions attempt to internalize the cost of appropriation of commons by adding this to the cost of private resources. In this paper we will show that this approach has some fundamental weaknesses.*

*To overcome these shortcomings, we have proposed to segregate the management of private and shared resources by using non-scalar numbers for the underlying economic signals of money and price. We prove that the resulting non-scalar economy always reaches a macro-economic equilibrium point where the level of appropriation of commons by the society remains steady.*

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## 1. Introduction

The world economy is now confronted with the scarcity of open access and non-excludable resources such as the atmosphere and the oceans. While the current market economy is reasonably effective in managing scarce resources that are *privately owned*, it lacks an inbuilt mechanism to manage such *shared* resources. In particular, the cost of appropriation of these commons is *external*, which results in over-exploitation. Most governments are now introducing incentive-based schemes, such as effluent taxes or tradable permits, to confront the polluters with this negative externality [1][2].

In these schemes, the cost of appropriation of commons is *added* to the cost of private resources. Unfortunately, by lumping these costs together, a fundamental weakness is created because the private and shared resources cannot be differentiated or managed independently. In the course of market transactions, as one product is used as an input for another, the contribution of shared resources is progressively obfuscated and swamped by other costs. The system, therefore, does not maintain accurate information, and the signals and incentives within the economy are rather blunt and cannot be targeted at specific objectives. In addition, damage to shared resources is *cumulative*, and the system should possess some ‘memory’ to account for this. However, by mixing the costs of private and shared resources, it would be difficult or impossible to keep an accurate account of the appropriation of commons in isolation, hence, we would be forced to use the *same* management model for both.

Development of a market economy that can manage private and shared resource independently would be, in our view, a more effective approach. We propose to achieve this segregation using non-scalar numbers for the underlying economic signals of money and price [3]. In this *non-scalar economy*, money and price are represented by a duple such as  $\mathbf{x} = (x_1, x_2)$ . The first component  $x_1$  is responsible for the management of privately owned resources and is called the *Private Resource Component* (PRC). The second component  $x_2$ , called the *Shared Resource Component* (SRC), manages the scarcity of shared resources of interest.

We have proposed *three core design elements* for the non-scalar economy so that both private and shared resources are efficiently allocated. This paper aims to provide an analysis of the non-scalar economy with respect to two key issues:

- 1- We demonstrate that the proposed non-scalar economy achieves a macro-economic equilibrium, in which the level of *SRC in circulation remains steady*. This equilibrium level can be controlled and is set only at the global level as a single global target.
- 2- The second contribution of this paper is to show that the proposed design of the non-scalar economy would guarantee the *accuracy of price signal with respect to the SRC*. In other words, rational economic agents would ensure that the SRC of the price neither underestimates nor overestimates the actual level of appropriation incurred.

The remainder of this paper is organized as follows: Section 2 provides a brief overview of the non-scalar economy and its underlying design. Section 3 discusses the macro-economic equilibrium point at the global and national levels. Section 4 develops a micro-economic model for all the influences on the pricing decision of a firm to demonstrate price signal accuracy. Concluding remarks are presented in Section 5.

## 2. A Brief Overview of the Non-Scalar Economy

The rules that govern the behaviour of players in the ‘game of economy’ are tacitly encoded and enforced in the underlying mathematical fabric of the system. For the non-scalar economy, we have incorporated our objective using three core elements in its design:

1. A new definition for the purchase and sell operations,
2. The addition of a new constraint associated with shared resources, and
3. The ability to tighten this constraint in accordance with the diminishing capacity of shared resources because of subtractibility.

This section briefly introduces these design choices and some of their ramifications. We focus only on those concepts that are relevant to the aim of this paper.

### 2.1 Design Element 1: Purchase and Sell Operations

In the current economy, the purchase operation is a *subtraction* of price from the tendered money. For example, if the buyer’s money before the purchase is  $\$x$  and the price of the desired good is  $\$a$ , the cash register performs a subtraction operation  $y = x - a$ , where  $y$  is the remaining money after the purchase. The purchase can take place provided  $y \geq 0$ .

Our proposed purchase operation in the non-scalar economy is *not* a subtraction, but remains mathematically simple. Let us assume that the buyer’s money before the purchase is a duple such as  $\$x = (x_1, x_2)$ . The price of the desired good is another duple such as  $\$a = (a_1, a_2)$ . In order to obtain  $\$y = (y_1, y_2)$ , which is the money left over after the purchase, the cash register in the non-scalar economy uses the purchase operation  $y = x \oplus a$ , where:

$$(y_1, y_2) = (x_1, x_2) \oplus (a_1, a_2) \equiv (x_1 - a_1, x_2 + a_2), \text{ provided the result is valid money} \quad (1)$$

The operand  $\oplus$  denotes the purchase operation of money  $x$  on price  $a$ . As can be seen,  $y_1 = x_1 - a_1$ . In other words, *private resources receive the same treatment* as in the current economy. However,  $y_2 = x_2 + a_2$ , which means that buyers *receive* and *accumulate* their share of the appropriation of commons via purchases.

The *sell* operation in the non-scalar economy is the reverse of purchase. In the above example, the seller’s SRC would increase by  $a_1$  and his/her SRC would decrease by  $a_2$ . Denoting the seller’s money before and after the sale by  $z$  and  $w$  respectively, the sell operation is:

$$(w_1, w_2) = (z_1, z_2) \odot (a_1, a_2) \equiv (z_1 + a_1, z_2 - a_2), \text{ provided the result is valid money}$$

### 2.2 Design Element 2: Scarcity constraint associated with shared resources

The scalar purchase  $y = x - a$  can only take place if  $y \geq 0$ . This restriction on the buying power is a reflection of the scarcity of private resources ensuring that “I cannot consume more than I have

earned". We carry this constraint forward to the non-scalar economy. In other words, in Equation (1) above, a necessary (but not sufficient) condition for  $\mathbf{y} = (y_1, y_2)$  to be valid money is that  $y_1 \geq 0$ .

We now need to develop a scarcity constraint for the shared resources. Note that the SRC of an economic agent *progressively increases* because of the consumption of goods. This mimics the accumulation of the corresponding pollution in the real world. The constraint on the SRC should somehow enforce a limit on this growth. But it is not appropriate to impose an arbitrary upper limit on the magnitude of the SRC of each agent. Therefore, for every money  $\mathbf{x} = (x_1, x_2)$ , we propose to restrict a *function* of the SRC called the *Shared Resource Index* (SRI) of  $\mathbf{x}$ , denoted by  $I(\mathbf{x})$  as defined below:

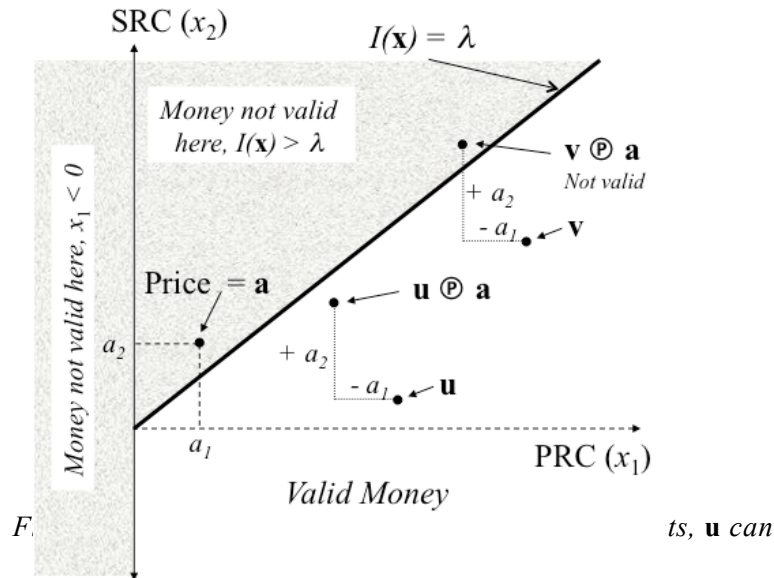
$$I(\mathbf{x}) \equiv \begin{cases} \frac{x_2}{x_1 + x_2} & \text{if } x_2 > 0 \\ 0 & \text{if } x_2 \leq 0 \end{cases} \quad (2)$$

Clearly, the SRI of any  $\mathbf{x}$  is confined between 0 and 1, and as  $x_2$  increases, the SRI approaches 1. Our proposed scarcity constraint for shared resources is *an upper limit* on the value of SRI. This limit is called the *SRI Threshold* and is a macroeconomic parameter set for *each country*, and therefore imposed on all transactions therein, as described later.

We can now formally define the condition of validity of money in the non-scalar economy as stipulated in Equation (1), which constitutes the second element of our design:

**Definition I:** In the non-scalar economy, money  $\mathbf{x} = (x_1, x_2)$  is *valid* provided  $x_1 \geq 0$  and  $I(\mathbf{x}) \leq \lambda$ , where  $\{\lambda \in \mathbb{R} : 0 < \lambda \leq 1\}$  is the SRI Threshold imposed on  $\mathbf{x}$ .

Consequently, the non-scalar purchase operation  $\mathbf{y} = \mathbf{x} \oplus \mathbf{a}$  can only take place if  $\mathbf{y}$  satisfies both



conditions of validity specified in Definition I above. Otherwise,  $\mathbf{x}$  *does not have sufficient buying power* to purchase this good. This is shown in Figure 1, where  $I(\mathbf{x}) = \lambda$  is a solid line in the two-dimensional plane created by the two components of money. Valid money is only defined in the region below and including this line. The purchase operation  $\mathbf{u} \oplus \mathbf{a}$  can proceed in Figure 1, because the result remains in the 'valid money' region. However,  $\mathbf{v}$  cannot afford  $\mathbf{a}$ , because the SRI of  $(\mathbf{v} \oplus \mathbf{a})$  exceeds  $\lambda$ .

If  $\lambda = 1$ , the scarcity constraint of shared resources is at infinity, i.e., shared resources are considered abundant, and the non-scalar economy is ‘reduced’ to the current economy.

**Marginal Cost of the SRC:** In the scalar purchase  $y = x - a$ , the buying power of money  $x$  is restricted because of the scarcity constraint  $y \geq 0$ . If we define the buying power of  $x$  as “the number of items at the unit price of \$1.00 that can be purchased by \$ $x$ ”, then the buying power is trivially equal to the magnitude of  $x$ . With non-scalar money, an analogous definition for the buying power would be: “the number of items at the unit price of  $\mathbf{a} = (1.00, 0.00)$  that can be purchased by  $\mathbf{x} = (x_1, x_2)$ ”. With this definition, it can be observed in Figure 1 that if  $x_2 \leq 0$ , the buying power would be equal to  $x_1$ . However, if  $x_2 > 0$ , the buying power of  $\mathbf{x}$  would be *less* than  $x_1$  and can be obtained by:

$$B_\lambda(\mathbf{x}) = x_1 - \left( \frac{1 - \lambda}{\lambda} \right) x_2$$

Here,  $B_\lambda(\mathbf{x})$  denotes the buying power of  $\mathbf{x}$ , where  $\lambda$  is the SRI Threshold imposed on  $\mathbf{x}$ . Assuming that  $\lambda$  is fixed, we can now quantify the impact of the SRC on the buying power of money:

$$\frac{\partial B_\lambda(\mathbf{x})}{\partial x_2} = -\rho, \text{ where } \rho \equiv \frac{1 - \lambda}{\lambda}, \quad x_2 > 0 \text{ and } \lambda \text{ is a constant.} \quad (3)$$

We refer to parameter  $\rho$  in the above expression as the *marginal cost of the SRC*. This means that for every extra dollar in the SRC, the buying power of money is reduced further by  $\rho$  dollars. However, unlike pollution taxes or the cost of permits, the presence of the SRC neither reduces one’s private resources nor provides revenue for some other entity. This is because the ownership of the SRC, *which is the source of buying power*, has not changed hands, although some of the buying power is no longer accessible. We say that the presence of the SRC *locks up* some of the buying power of money. By reducing every dollar of SRC,  $\rho$  dollars of buying power would be unlocked until the SRC becomes zero and the buying power reaches its maximum value of  $x_1$ . In [3] we showed that this property results in the creation of an environmental sector, which provides ‘environmental goods’ for sale. The environmental goods are produced by activities that restore the shared resource to its original condition, similar to today’s offset schemes. The SRC of the price of environmental goods is *negative*. Therefore, purchase of such goods would reduce the buyer’s SRC and unlock some of the latent buying power in his/her wealth.

**Accumulation of the SRC:** Figure 2 shows the accumulation of the SRC in the economy of a country such as Australia. The SRC enters the circulation through:

- Direct appropriation of shared resources by Australian people and firms, where *new* SRC is created and transferred to the appropriator, and
- Import of goods from overseas, where SRC is transferred to the importer via purchase.

The SRC then *circulates* in the economy via the operations of purchase and sell. For every dollar of

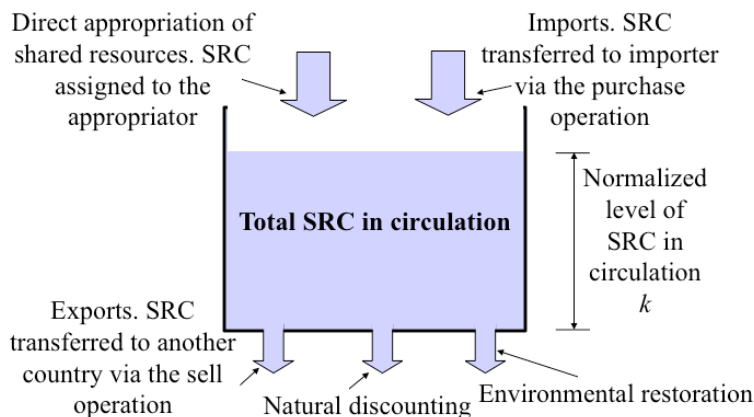


Figure 2: Accumulation of the SRC in the non-scalar economy

SRC in the economy,  $\rho$  dollars of buying power is locked up. Initially, this impact is small, but as time goes on, more SRC enters circulation. This accumulation mirrors the real environmental impact of the Australian economy on shared resources. If the accumulation were not stopped, the economy would eventually come to a grinding halt; a reflection of the fact that the natural capacity of shared resources would be exhausted. Investment in pollution abatement and a more judicious choice of imports might reduce the rate of *in-flow* of SRC to Australia. There are also three avenues for SRC to flow *out* of circulation as shown in Figure 2:

- Natural discounting is applied to the whole economy at regular intervals to mimic the regeneration capability of shared resources,
- The export of goods to other countries, where the SRC is transferred to overseas buyers via the sell operation, and
- The efficient production and sale of environmental goods by the environmental sector.

It will be shown later that the economy *always* reaches an equilibrium point where the total SRC in circulation stabilizes at a particular level. This equilibrium level is a function of  $\lambda$ .

### 2.3 Design Element 3: Tightening of the shared resource constraint

A fundamental difference between private and shared resources is that one's appropriation of a shared resource affects the capacity available to others. The marginal cost of further appropriation would then depend on the size of the remaining capacity. Therefore, the marginal cost of appropriation of commons is *not* a constant and depends on the *history* of appropriation. For example, a particular polluting activity may indeed have *negligible* cost in a *clean* environment, but if it occurs after centuries of pollution and as part of a large community of polluters, its impact may no longer be ignored. The marginal cost of appropriation, therefore, must be an *increasing function* of the total level of appropriation by the economy. The non-scalar economy provides the necessary tools to achieve this goal because: (i) the economy keeps track of the appropriation of shared resources automatically and accurately, and (ii) the marginal cost of the SRC can be controlled with a single parameter.

Therefore, our third design element is:

The SRI Threshold ( $\lambda$ ) imposed on each country must be a *monotonically decreasing function of the normalized level of SRC in circulation*.

The normalized SRC in circulation for a country is shown as  $k$  in Figure 2. The normalization may typically be on a per-capita basis but may also consider other factors such as the development stage of the economy.

The consequence of our third design element is that, as more SRC is accumulated,  $\lambda$  is decreased, signifying that the resource capacity has been diminished and the constraint associated with shared resources is now tighter. Reduction of  $\lambda$ , in turn, increases the marginal cost of SRC  $\rho$  for *everyone*. Therefore, SRC build-up is a *public bad*, both economically and in the real world. In the long run, the accumulation of SRC in the economy has two effects: (i) it expands the market available to the environmental sector, and (ii) the resulting increase in  $\rho$  would improve the returns of, and prompt cooperative incentives to invest in, the environmental sector.

## 3. Macro-Economic Equilibrium

Let us consider a non-scalar economy with a monotonically decreasing function for the SRI Threshold imposed on it. We now show that such an economy will reach an equilibrium state if at all feasible, that is, if the marginal cost is finite. The equilibrium point is defined as *a state of economy when the per-capita SRC in circulation is held at a constant level*. The first order condition for this point is that the marginal benefit of reducing per-capita SRC in circulation becomes equal to its marginal cost. Below, we will derive an expression for these quantities.

### 3.1 Marginal benefit of reducing per-capita SRC in circulation

The normalized level of SRC in circulation for a country is denoted by  $k$  in Figure 3. The SRI Threshold  $\lambda$  should be a monotonically decreasing function of  $k$  based on our third design element. In practice, this may be in the form of a step function, as shown in Figure 3, with re-evaluation and imposition of a new threshold occurring at regular intervals such as on a quarterly basis. To simplify analysis, let us assume a continuous case for which  $\lambda$  is decreased linearly as a function of  $k$  at a particular rate of  $\alpha$ . That is,

$$\lambda = 1 - \alpha k; \quad 0 \leq k < \frac{1}{\alpha} \quad (4)$$

The parameter  $\alpha$  in the above equation determines how aggressively the SRI Threshold is adjusted (lowered) in response to rising  $k$ , which in turn determines the length of time before the equilibrium is reached as well as the actual equilibrium point.

Let the normalized cost of SRC as a function of  $k$  be  $L(k)$ . Given that everyone's SRI Threshold is  $\lambda$ , then the per-capita cost of SRC, equal to the locked up buying power under  $\lambda$ , would be:

$$L(k) = \frac{1 - \lambda}{\lambda} k$$

Hence, the *marginal benefit of reducing*  $k$ , denoted by  $\hat{\rho}$  is obtained by:

$$\hat{\rho} \equiv \frac{\partial L}{\partial k} = \frac{\alpha k(2 - \alpha k)}{(1 - \alpha k)^2} = \frac{1 - \lambda^2}{\lambda^2} \quad (5)$$

For  $0 < \lambda < 1$ ,  $\hat{\rho}$  is *always greater* than the  $\rho$  derived via Equation (3). In other words, the marginal cost of accumulation of SRC in one's country is actually *higher* than the marginal cost of accumulation of SRC in one's own wealth. This disparity, *which is the foundation of cooperative incentives in the non-scalar economy*, arises because any reduction in  $\lambda$  would increase  $\rho$  and this change would be applied, *retrospectively*, to all the SRC accumulated in the past. In other words,  $\rho$  is the short run marginal cost, which ignores the full impact of SRC accumulation on the country's economy in the long run.

### 3.2 Marginal cost of reducing per-capita SRC in circulation

As shown in Figure 2, the level of per-capita SRC in circulation  $k$  can be reduced by: (i) reducing

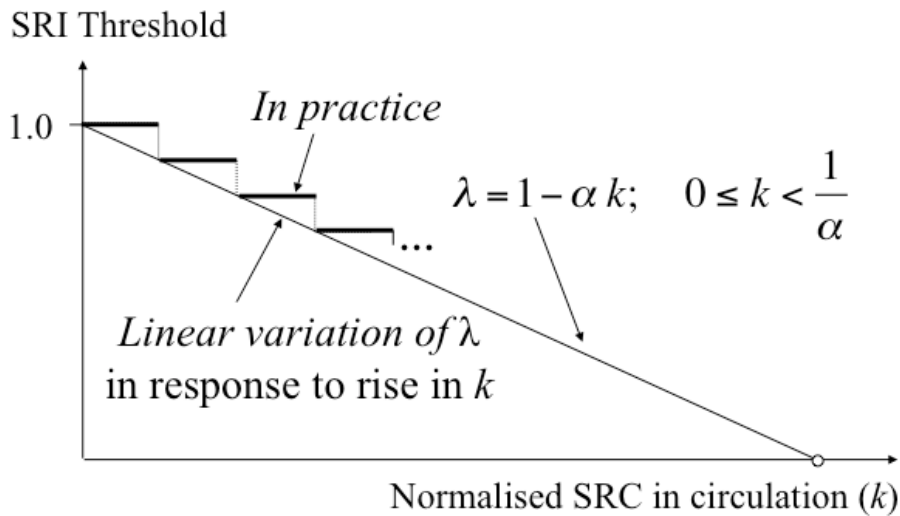


Figure 3: Variation of SRI Threshold in response to accumulation of SRC in the economy



the rate of SRC in-flow to the economy by judicious choice of imports and abatement activity, and (ii) removing some of the SRC from circulation by exports and environmental restoration. The combination of abatement and environmental restoration could be viewed as an *environmental good* offered by the environmental sector with a *total* price of  $(e_1, -e_2)$ . This price indicates the efficiency of the environmental sector in using privately owned resources to the value of  $e_1$  to reduce the SRC by  $e_2$ . Let us assume that environmental goods to the total value of  $(e_1, -e_2)$  are required to keep the per-capita SRC in circulation at a particular value  $k$ . Then the marginal cost (in terms of lost buying power) of reducing  $k$  would be  $\frac{\partial e_1}{\partial k}$ .

The shape of this function cannot be ascertained at this stage and would depend on the efficiency of the environmental sector, level of competition, spending on R&D and advancements in technology. However, evaluation of this function would be quite feasible given the exact accounting information readily available when the non-scalar economy is operational. It is reasonable to assume that this is a *decreasing function* of  $k$  due to economies of scale and usually high establishment cost of infrastructure for abatement and provisioning. In addition, as  $k$  approaches infinity, less effort would be required to maintain the level of SRC in circulation.

### 3.3 The Equilibrium Point

Figure 4 shows the equilibrium points as the intersection points of a hypothetical marginal cost function with three possible marginal benefit functions with different values of  $\alpha$ . Clearly, the marginal benefit of reducing  $k$  grows to arbitrarily large values in response to accumulation of SRC.

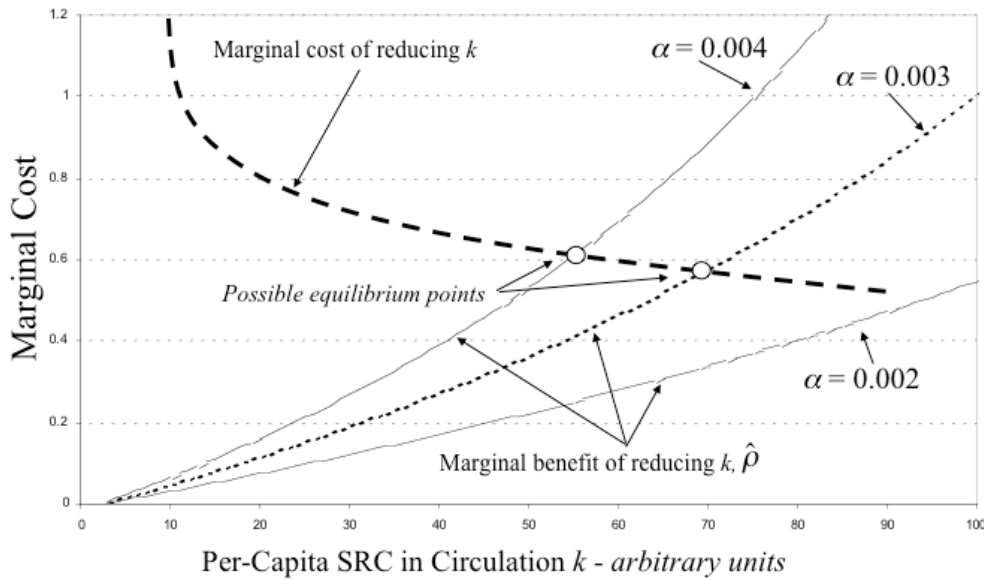


Figure 4: Equilibrium Points for different rates ( $\alpha$ ) of lowering SRI Threshold in response to rising  $k$

Hence, for any *finite* marginal cost and regardless of the shape of this function, there will be an equilibrium point.

At the equilibrium point, there is no further accumulation of per-capita SRC. This does not imply that improvements in the standard of living are stopped. It merely states that the environmental sector grows with the economy or is becoming more efficient in reducing the polluting side effects of generating wealth.

### 3.4 Equilibrium point at the national and global levels

For the management of private resources, an economy needs *external* institutions to regulate and enforce ownership rights. The shared resources are not owned by anyone, so property rights are not needed. However, the economic system requires one piece of information to be supplied from outside, namely *the total capacity of shared resources*. In the current schemes, this is achieved by negotiating a set of national targets for emissions. In the non-scalar economy, only a single global target for the total capacity is required and the market would determine the level of accumulated SRC in various countries based on the comparative advantages of each economy. To illustrate the point by a simple example consider Figure 5, which shows the SRC accumulation in the world economy (“import” and “export” flows of Figure 2 are now *internal* transactions at the global level). The world’s normalized SRC in circulation ( $k$ ) can be controlled by choosing a suitable model for the monotonic reduction of  $\lambda$ . A hypothetical set of such values is shown in the Figure. This *same* function is then applied to all participating countries. Given the comparative advantages of different economies, such as the strengths of their environmental sector, each country will have a specific marginal cost curve and will achieve its own equilibrium point. The competitive and cooperative incentives among participating economies would eventually lead to the desired global equilibrium.

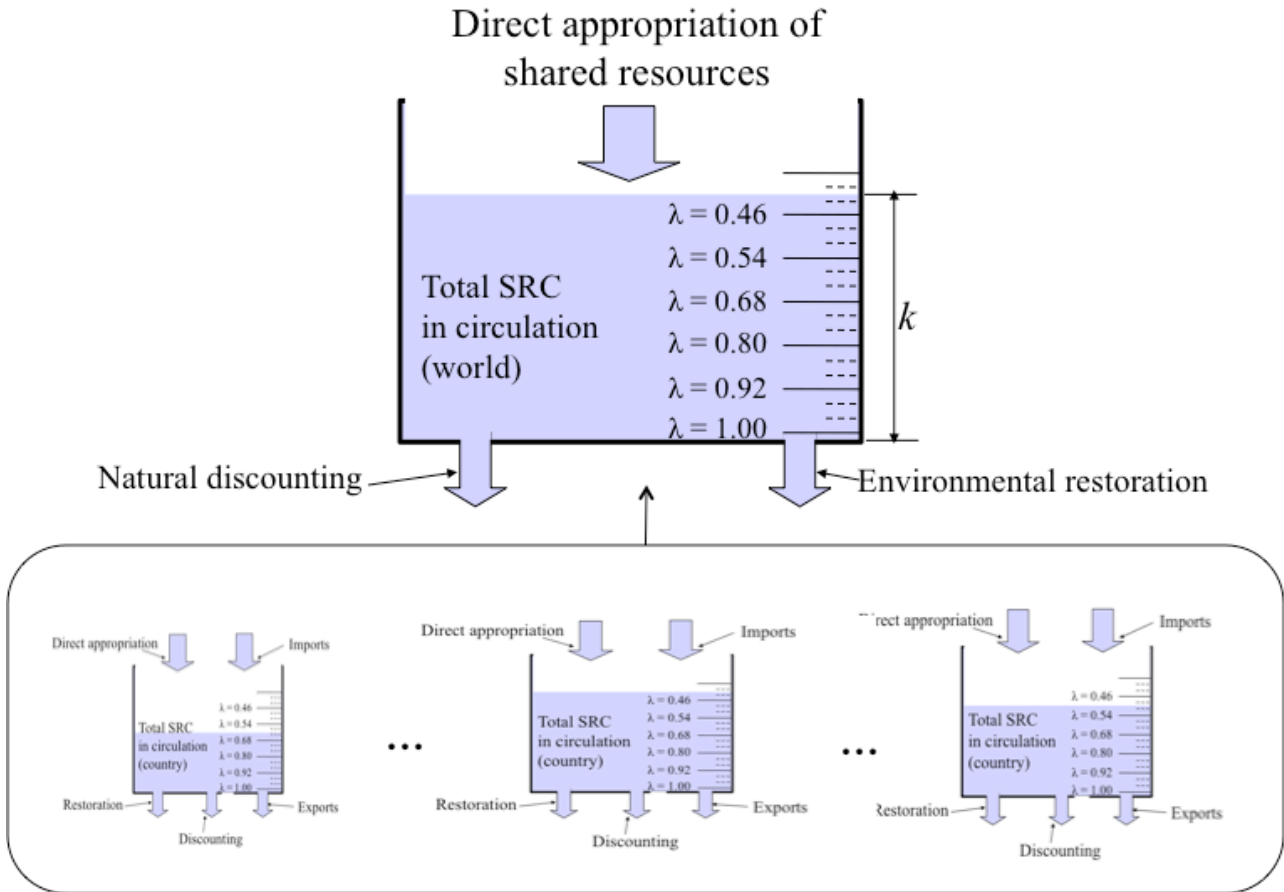


Figure 5: SRC accumulation in the world and national economies and an example assignment of  $\lambda$  based on  $k$

## 4. Price Signal Accuracy

Figure 6 shows the SRC accumulation in a firm in the non-scalar economy. A firm may be *directly* engaged in the appropriation of global commons, such as the discharge of waste into the atmosphere or oceans. This direct appropriation is accounted for by the creation of *new* SRC, which is transferred to the firm. Often more importantly, the firm also utilises shared resources *indirectly*,

through the purchase of inputs, such as electricity, raw materials, transport services and the like. The SRC associated with these inputs is transferred to the firm via the purchase operation.

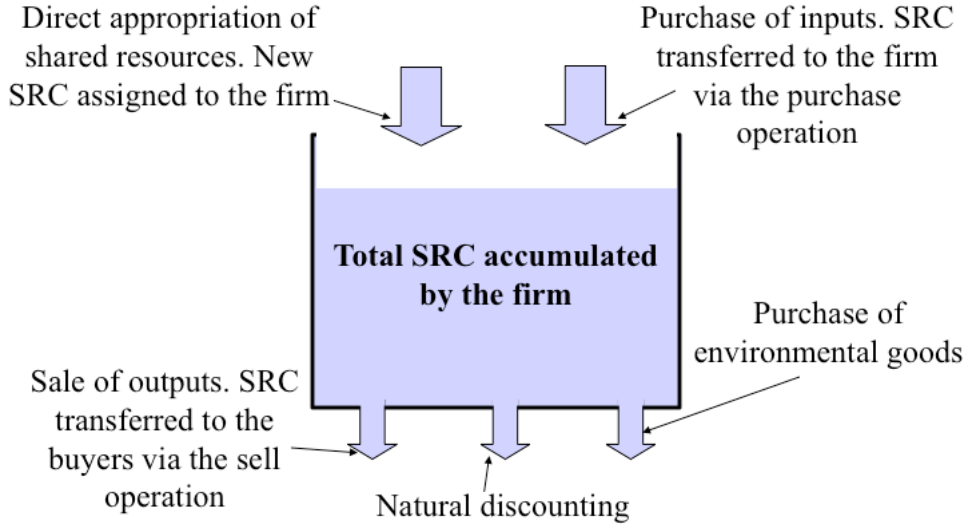


Figure 6: Accumulation of SRC in a firm

Some of the accumulated SRC may be transferred to the firm's customers via the sell operation. If the SRC of the price is properly set, there would be no build-up of the SRC in the long run and the price signal would be *accurate* with respect to the appropriation of commons. However, the firm has complete freedom in setting the price. For example, the firm can substitute SRC for PRC or vice versa, because by reducing one dollar from the SRC and adding  $\rho$  dollars to the PRC of price, it would receive the same buying power when the product is sold.

Nevertheless, we believe that the current design of the non-scalar economy is *sufficient* to ensure that, with respect to the SRC, the price signal is *costly-to-fake*. To illustrate this point, in this Section we develop a model for all the influences that would have an impact on the pricing decision of a firm, incorporating the effects of both competitive and cooperative incentives.

#### 4.1 Perception of price

Let us first develop a *scalar* quantity to measure how 'expensive' a two-dimensional price appears to buyers. One possible approach is to use the amount of buying power lost due to purchase for this purpose. Let us call this the '*magnitude*' of the price. Using the definition of buying power developed in Section 2, it can be shown that the magnitude of price  $\mathbf{a} = (a_1, a_2)$  for a buyer with an SRI Threshold of  $\lambda$  would be:

$$M_\lambda(\mathbf{a}) = a_1 + \rho a_2 \quad (6)$$

Which means that:

$$\frac{\partial M_\lambda(\mathbf{a})}{\partial x_2} = \rho, \quad \text{where } \rho \equiv \frac{1 - \lambda}{\lambda}$$

This result produces the same marginal cost of SRC ( $\rho$ ) as Equation (3). Therefore, for every additional dollar in the SRC of the price, the buyer perceives the item to be  $\rho$  dollars more expensive, purely in terms of loss of buying power following purchase. Incidentally, the same expression for the magnitude applies to sellers. That is, by selling an item priced at  $\mathbf{a} = (a_1, a_2)$ , the seller *gains*  $M_\lambda(\mathbf{a})$  of buying power, where  $\lambda$  is the SRI Threshold *imposed on the seller*. If the buyer and seller are under the same SRI Threshold, then the amount of buying power lost and

gained during a transaction will be the same. In general, however, their SRI Thresholds may be different and the two parties would have dissimilar views about the exchanged buying power. This property is of unique importance in the non-scalar economy, and as discussed later, is relevant even when buyers and sellers reside in the same country and *nominally* under the same SRI Threshold.

## 4.2 Pricing options available to the firm

For now, let us ignore the impact of competition and focus on a firm in isolation. In Figure 7, the point  $\hat{\mathbf{a}} = (\hat{a}_1, a_2)$  represents the average total cost incurred by producing a unit of output. The horizontal line through this point, labelled AP, is the locus of all *accurate* prices (i.e.,  $\text{SRC} = a_2$ ). By setting the price at point  $\mathbf{a} = (a_1, a_2)$ , the sale of each unit of output returns  $a_1 - \hat{a}_1$  worth of buying power regardless of the SRI Threshold  $\lambda_f$  imposed on the firm. However, the same buying power would be returned if the price was set along the line labelled IMF, which is the line of constant magnitude based on  $\lambda_f$  that goes through  $\mathbf{a}$ . Any price point on this line above AP *over-estimates* the SRC incurred by production, but has less PRC than  $a_1$ . Therefore, some of the buying power returned to the firm in this case is *indirect*, due to the reduction of accumulated SRC in the past. On the other hand, any price on IMF below the AP line *under-estimates* the SRC incurred and its PRC is greater than  $a_1$ . Under this condition, the firm accumulates SRC and, for every dollar of SRC,  $\rho$  dollars of additional PRC will have to be acquired, which remains locked up. Eventually, the accumulated SRC either has to be transferred to future customers or returned as a dividend to shareholders (who are likely to be displeased).

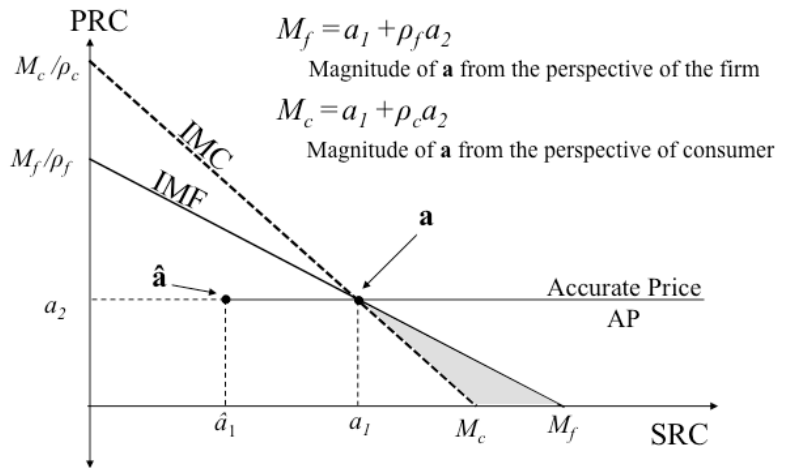


Figure 7: Perspectives on prices by the firm and its customers

Of course, the buying power of money and the magnitude of price, being *scalar quantities*, cannot embody all the information contained in the non-scalar money and price. Some of the prices on the IMF line, such as price points close to the PRC and SRC axes, will not be credible in the market. A more accurate model could include a general concept of utility, which would result in a (non-linear) curve similar to the *indifference curve* (locus of  $U_{x1} dx_1 + U_{x2} dx_2 = 0$ ) to better represent the perspectives of consumers and firms. For the purpose of our current discussion, however, the exact shape is not critical and the above model is sufficient.

## 4.3 Competitive pressures

The competitive market in the non-scalar economy is similar to today's free market model and therefore, the prices of a firm are affected by competition and demand. In the current economy, all else being equal, firms can only distinguish themselves by the magnitude of their prices. However, the *magnitude* of non-scalar prices, as defined in Equation (6), may not be sufficient to model the attitude of consumers. For example, assume that  $\lambda = 0.50$  ( $\rho = 1.00$ ), then both prices  $\mathbf{a} = \$ (90, 10)$  and  $\mathbf{b} = \$ (10, 90)$  will have the same magnitude of \$100. However, semantically, these represent very different signals. More importantly, purchase at these prices would also have *very different economic consequences for the consumer*.

A unique competitive pressure on the firm's prices in the non-scalar economy is caused by the *environmental sector*. To illustrate this point, let us assume that the equilibrium price of a unit of an environmental good is  $\mathbf{e} = \$ (e, -1.00)$ . By purchasing this good, the buyer reduces \$1.00 of his/her

accumulated SRC and pays \$ $e$  of PRC to compensate the environmental firm for the private resources used in its operation. The net buying-power gain for the buyer is  $\rho - e$ . Given that environmental goods are not consumables, rational agents would only consider buying these if, among other things,  $\rho > e$ . Now consider the case when the firm wishes to reduce the SRC of its price by \$1.00 below the accurate value of  $a_2$  on the IMF line in Figure 7. The new (inaccurate) price would be  $(a_1 + \rho, a_2 - 1.00)$ . For the consumer, purchase at this price is equivalent to purchase at  $(a_1, a_2)$  followed by the purchase of an environmental good priced at  $(\rho, -1.00)$ . Similarly, setting the SRC of the price \$1.00 above the actual value on the IMF line results in  $(a_1 - \rho, a_2 + 1.00)$ . For the firm, this is equivalent to selling at  $(a_1, a_2)$  and then buying an environmental good at  $(\rho, -1.00)$ . In both cases, the price of the environmental good is *not competitive* because  $\rho > e$ . So the firm is forced to consider a *higher effective SRI Threshold* compared to its actual (so that  $\rho_f \approx e$ ) if it wants to set inaccurate prices, which reduces the return to the firm.

#### 4.4 Cooperative pressures

The short run and long run marginal costs of SRC were derived in Equations 3 and 5 respectively. The long run cost is always higher because the accumulation of SRC in one's country results in a reduction of  $\lambda$  for everyone. But although the changes in  $\lambda$  are imposed on all, the impact is particularly great for those who possess significant levels of SRC.

To illustrate by a simple example, consider a firm in Australia that accumulated \$10M of SRC in the previous quarter as a result of inaccurate prices. Also assume that, during the previous quarter,  $\lambda = 0.65$  ( $\rho = 0.54$ ), so the firm had to increase the PRC of its prices accordingly to receive an extra \$5.4M of PRC, which is now locked up against the accumulated SRC. Now suppose that, as a result of accumulation of SRC in Australia (some of which might well have been contributed by this firm), the SRI Threshold of Australia is lowered in this quarter to, say,  $\lambda = 0.62$  ( $\rho = 0.61$ ). This will cause an increase in the locked-up buying power of this firm from \$5.4M to \$6.1M. Had the firm adopted accurate prices, its buying power would not have been affected by this change in  $\lambda$ . In other words, while the firm and its customers were nominally under the same SRI Threshold of  $\lambda = 0.65$  in the previous quarter, the marginal cost of changes in  $\lambda$  was higher for the firm compared with its customers, *given its propensity to accumulate more SRC*.

To illustrate this point, assume that the total wealth owned by an agent at a given time is  $\mathbf{x} = (x_1, x_2)$ . Then the marginal of cost of changes in  $\lambda$  using the expression for buying power derived in Section 2 will be:

$$\frac{\partial B_\lambda(\mathbf{x})}{\partial \lambda} = \frac{1}{\lambda^2} x_2$$

That is, the marginal cost of changes in  $\lambda$  is proportional to the level of accumulated SRC. Consequently in the above example, although a typical customer could have used the short run marginal cost ( $\rho$ ) based on the current value of  $\lambda = 0.65$ , such a strategy would be quite myopic for the firm.

Therefore, to be prudent, the firm that intends to set inaccurate SRC for its prices, would have to include the possibility of a reduction in  $\lambda$  and adopt an effective threshold in the calculation of its price, which is lower than the current value of  $\lambda$ . That is why, in Figure 7, the firm's SRI Threshold is shown to be lower than its customers' (i.e.,  $\lambda_f < \lambda_c$ ) even when they are in the same country. In this case, any price below the AP line would actually appear more expensive to customers (shaded triangle in Figure 7).

#### 4.5 Summary

We established in the previous subsections that *competitive* pressures would force the firm towards the case of increasing its effective SRI Threshold (i.e.,  $\lambda_f > \lambda_c$ ), where  $\lambda_f$  and  $\lambda_c$  are effective SRI

Thresholds felt by the firm and its customers respectively. On the other hand cooperative pressures require reduction of the effective SRI Threshold (i.e.,  $\lambda_f < \lambda_c$ ). Hence, when both these pressures are present, any deviation from accurate pricing will have a significant market cost for the firm.

## 5. Conclusions

By adding the cost of appropriation of commons to the cost of private resources several weaknesses are created in the economy:

- 1- There is a one-to-one correspondence – dollar for dollar – between private and shared resources. To affect a change in behaviour, the assigned cost to the commons should be comparable *in magnitude* to the cost of private resources used for abatement or restoration. This requirement is artificial, unrealistic and difficult to justify. Evaluation of worth of commons outside a proper market system that deals with these resources effectively is fraught with difficulty and a source of controversy. In addition, this cost is not a constant and depends on the existing context (level of appropriation) and the available capacity of the resource.
- 2- The economy is not empowered to keep accurate account of appropriation of commons because by mixing the two categories of resources, the contribution of shared resources is obfuscated.
- 3- The price signal is blunt and lacks suitable information for the consumer with respect to level of appropriation of commons incurred in production of good and services.
- 4- Damage to the environment may create revenue for some agents. For example, the government may receive revenue through taxation or sale of permits. This could be a source of perversion of incentives. In reality, the environmental degradation is a public bad and the economy should model it as such.

This paper demonstrates that the proposed non-scalar economy addresses these issues effectively. In the same order above:

- 1- The dollar value assigned to a unit of shared resources is not critical for the proper functioning of the economy. Indeed, this can be viewed as choosing a currency unit: as long as it is applied consistently the actual magnitude of the unit is immaterial. The real cost of the appropriation of commons felt by the economic agents is the product of this unit and the marginal cost of the SRC which is dependent on the context of accumulation of pollution.
- 2- Non-scalar economy keeps an accurate account of the appropriation of commons and has an effective mechanism to modify the marginal cost of SRC in response to the accumulation of pollution caused by the economy.
- 3- We have demonstrated that the price signal is accurate, that is, the SRC of the price can be relied upon to accurately represent to total appropriation of shared resources in producing the good or service. Moreover, the economic consequences of an inaccurate price are borne by the firm.
- 4- There is no transfer of wealth and no revenue generation as a result of creation of new SRC or transfer of SRC from one agent to another in the course of economic transactions. No one benefits from accumulation of SRC, so there will be a strong cooperative incentive to manage such accumulation in the economy.

### 5.1 Future work

We know from Coase theorem [4] that imposing the liability on the polluter (e.g., through taxation) may not always lead to an efficient outcome. Another distinction of the competitive market in the non-scalar economy is that the liability can be exchanged between the firm and its customers, which might allow a Coasian resolution to improve the outcome.

Given the various categories of commons, e.g., atmosphere and the oceans, in reality there will be a vector of SRC goods and it would be infeasible for every agent to face a vector of SRC prices.

Therefore there must be an aggregation to a single overall price. One possible approach is to maintain fixed relative prices among the various categories of commons. Any aggregation of prices leads to some loss of information and creates substitution opportunities that may be considered a weakness. Investigation of this issue will also be the subject of future publications.

## 6. References

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